Piscataway High School
AP Calculus AB
Summer Assignment 2023-24

Welcome to AP Calculus AB! This assignment is designed to help you review and practice the prerequisite skills necessary to succeed in calculus. The assignment is due **the first day of school**. It is important that you take time to understand and master these skills. **There will be a test on the topics covered within this assignment during the first week of school.**

You will need to use your graphing calculator to complete the last section of this assignment. All other sections should be completed without the aid of a graphing calculator, but you may utilize your calculator to check your answers. There are many resources available online to assist in using the graphing calculator. You can elect to borrow a calculator from the school or procure your own. If you are using your own graphing calculator you will need to familiarize yourself with its functionality prior to the start of the school year.

The assignment will be graded based on effort and accuracy. Work must be shown *in a neat and organized manner* to support your answer, and all work must be completed on a separate sheet of paper. Please group problems by section and include the section label. An attempt must be made to answer ALL problems. **Put your full name on the top right corner of EVERY page of your assignment**. You will need to utilize a scanner app to scan in all pages if not completed electronically, save your assignment as a **single pdf file,** and submit this file to the schoology group under the assignment titled, "AP Calculus AB Summer Assignment".

It is encouraged that you attend the AP Calculus AB summer institute. This is a 4-day course offered July 18-21 OR July 25-28 from 8-11 am at PHS. During this course we will review the prerequisite skills and complete the majority of the summer assignment. You can sign up for the course by filling out the google form that is on Schoology or linked in your welcome letter.

The summer assignment is **due by 7:15 am on the first day of school as a single pdf file submitted on Schoology**. I look forward to a great year getting to know all of you and doing a lot of fun calculus!

Sincerely,

Dr. Maurer

Dr. Amanda Maurer
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Section 1: Functions

Linear Functions
Skill: Write the equation of a linear function in point-slope form. In order to write a linear equation in point-slope form both the slope of the line, m, and one point on the line, (x_1, y_1), must be known (it does not need to be the y-intercept).

Point-Slope Form
$y - y_1 = m(x - x_1)$

Example: Write the point-slope form of the line given $m = 3$ and $(-1, 5)$.

Solution: $m = 3$ and $(x_1, y_1) = (-1, 5)$, therefore $y - 5 = 3(x - (-1))$

Write the equation of the line in point-slope form that satisfies the following conditions.

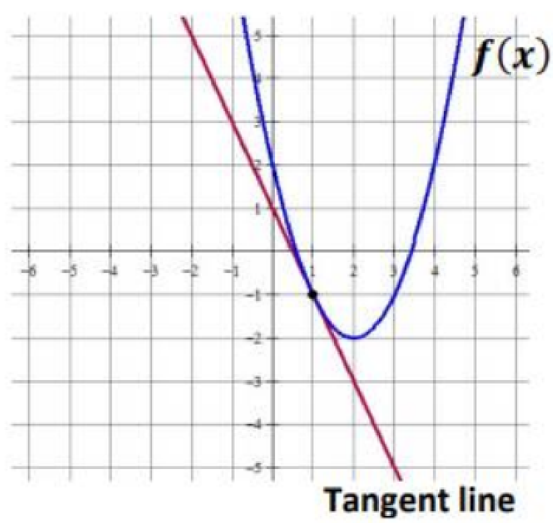
1) $m = 3$ through the point $(5, -2)$.

2) $(-2, 1)$ and $(-1, 4)$.

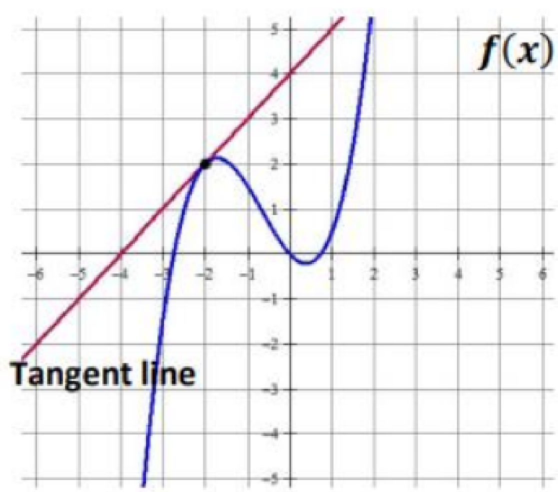
3) The line parallel to $y = 4x - 2$ and passes through the point (3, -4).

4) The line perpendicular to $x - 4y = 7$ and passes through the point (3, -4).

5) The line tangent to $f(x)$ at x = 1.



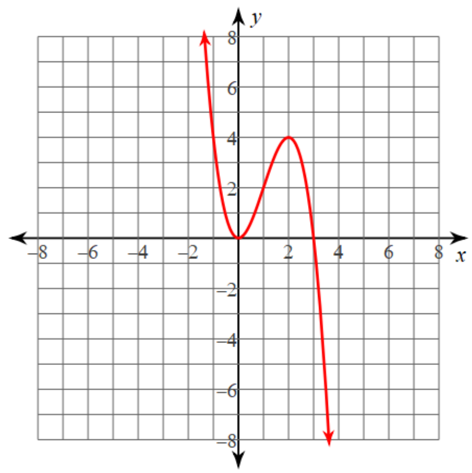
6) The line tangent to $f(x)$ at $x = -2$.



Attributes of Functions

Skills: Determine if a relation is a function. Find the domain and range, find intervals of increasing/decreasing, identify local and absolute maximum/minimum values.

Example: Identify attributes of the function $g(x)$ graphed on the right.



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Inc: $(0, 2)$ Dec: $(-\infty, 0]$ and $[2, \infty)$

Positive: $(-\infty, 3)$ Negative: $(3, \infty)$

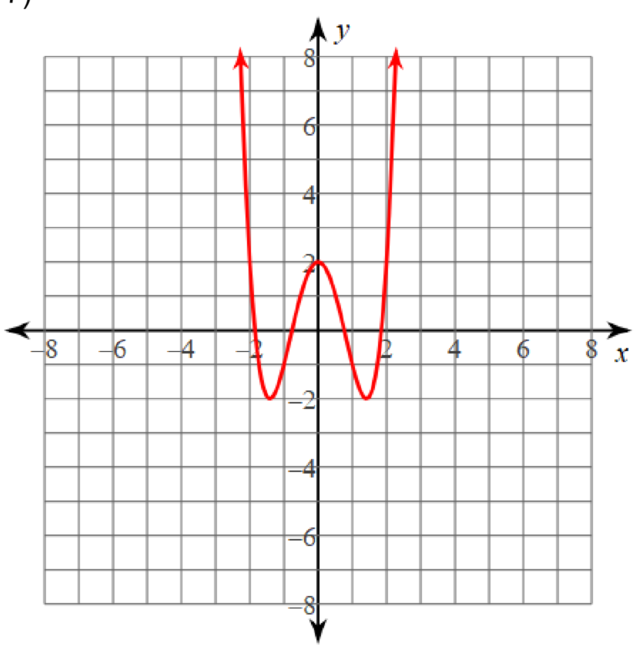
Relative Max: 4 (@ $x = 2$)

Relative Min: 0 (@ $x = 0$)

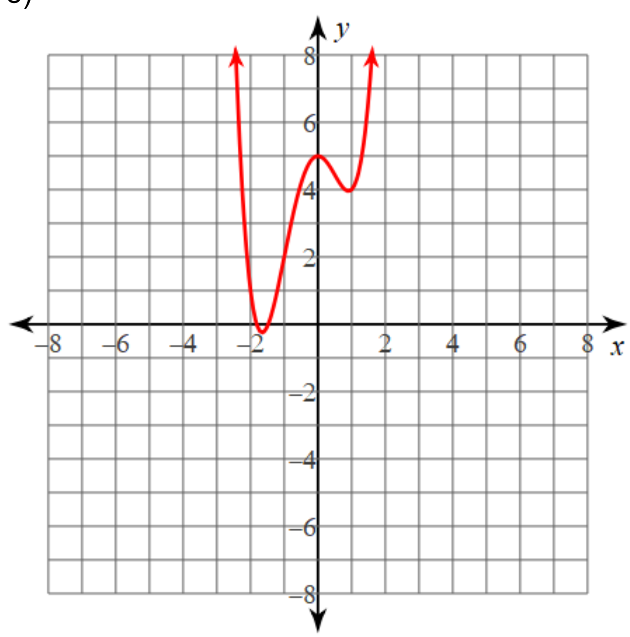
Notes: Max and Min are the outputs of the function (not the ordered pair).

For each graph below identify: domain, range, intervals of increasing and decreasing, where the function is positive (> 0) and negative (< 0) and all relative max and min values. Estimate the values based on the graph.

7)



8)



Graphing Functions

Skills: Graphs of common parent functions. Graph functions using knowledge of parent functions and transformations of functions.

For any parent function $f(x)$ the standard transformation form can be represented as follows:

Standard Transformation Form
$a \cdot f\left[b(x-h)\right]+k$

Example: Given $f(x)=x^2$ describe the transformations given by $1-f(2x+4)$.

Solution: First write the function in transformation form: $-f\left[2(x+2)\right]+1$
x^2 is being reflected over the x-axis with a horizontal compression of 2, a shift left of 2 units and shift up of 1 unit.

Graph each separately without a graphing application using your knowledge of the parent function and transformations. State the domain and range.

9) $y=\frac{1}{x-1}-3$

10) $y=\sqrt{-x}$

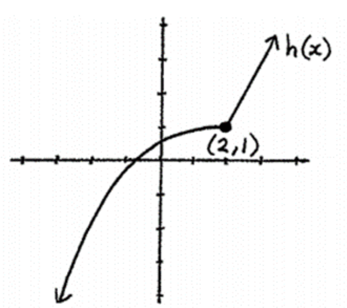
11) $y=2\sqrt{x+5}$

12) $y=1-(x+3)^2$

13) $y=|2x|+3$

14) $y=\frac{(x+1)^3}{2}+3$

15) Use the graph of $h(x)$ given below, then graph the following transformations.



Graph all separately or graph a-c on one graph (label each) and d-e on one graph (label each).

a) $h(x+1)$
b) $3h(x)$
c) $1-h(x)$
d) $-h(x)$
e) $h(-x)$

Graph each separately without consulting a graphing application. State the domain and range.

16) $y=\frac{x^2-x-6}{x-3}$

17) $y=\sqrt{25-x^2}$

18) $y=\left|x^3\right|$

19) $y=\sqrt{x^2}$

20) $y=\sqrt{|x|}$

21) $y=-\sqrt{-x}$

Even and Odd Functions

Even: Function is symmetrical about the y-axis. $f(x) = f(-x)$

Example: If function is even and $f(2) = -4$ then $f(-2) = -4$

Odd: Function has point symmetry about the origin (rotation of 180º). $f(-x) = -f(x)$

Example: If a function is odd and $f(2) = -4$ then $f(-2) = 4$

22) Knowing that $f(x)$ is an even function and $g(x)$ is an odd function, find $f(g(2))$ given $f(5) = -3$ and $g(-2) = 5$.

Determine, without a graphing calculator, if each function is even, odd, or neither. Provide evidence or an explanation to support your choice.

23) $y = (x-5)^2 + 1$

24) $y = \dfrac{5}{x}$

25) $y = \dfrac{x}{3+x^2}$

26) $y = \sin x$

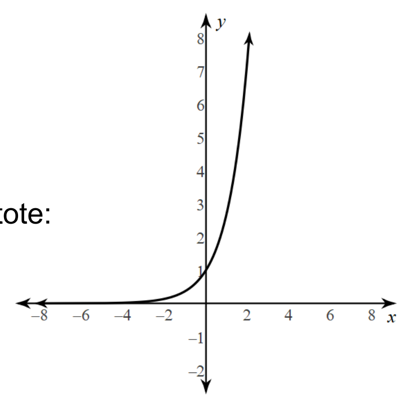
Graphing Exponential and Logarithmic Functions

Exponential Function ($y = b^x$)

Example: $y = e^x$

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Horizontal Asymptote: $y = 0$

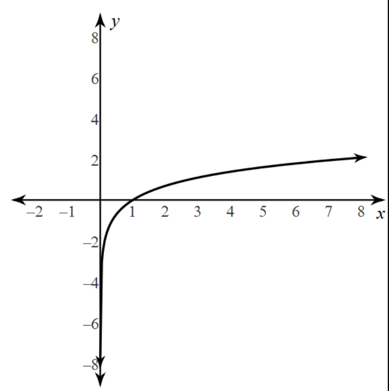


Logarithmic Function ($y = \log_b(x)$)

Example: $y = \ln x$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$



Graph each separately without consulting a graphing application. State the domain and range of each. You should be able to quickly sketch the graphs without using a calculator and without making an x-y table by using your knowledge of the parent function and transformations.

27) $y = -e^x$

28) $y = e^{-x}$

29) $y = |e^x - 1|$

30) $y = \ln(-x)$

31) $y = -\ln(-x)$

32) $y = \ln|x|$

Evaluating Functions
Skills: Evaluate a function (or expression involving a function) algebraically, graphically and numerically. Example: Evaluate $f(3a)$ for $f(x) = x^3 - 4x + 5$ Solution: Substitute $3a$ in for x so $f(3a) = (3a)^3 - 4(3a) + 5$ Then simplify, $f(3a) = 27a^3 - 12a + 5$

33) Use the following functions to evaluate each.

$$g(x) = e^{x+4} \qquad f(x) = x^2 - 5 \qquad j(x) = x - x^4$$

a) $j(-2)$

b) $f(3x^2)$

c) $g(f(x))$

d) $f(b+3)$

e) $g(-4) - j(f(3))$

f) $\dfrac{f(x+h) - f(x)}{h}$

34) Use the table below of $f(x)$ to evaluate: $f(-1)$, $f(f(3))$, $5 - f(5)$

x	-5	-1	3	5	6
$f(x)$	10	4	-5	-2	0

35) Use the graph of $f(x)$ on right:

a) State the domain and range.

b) Evaluate:

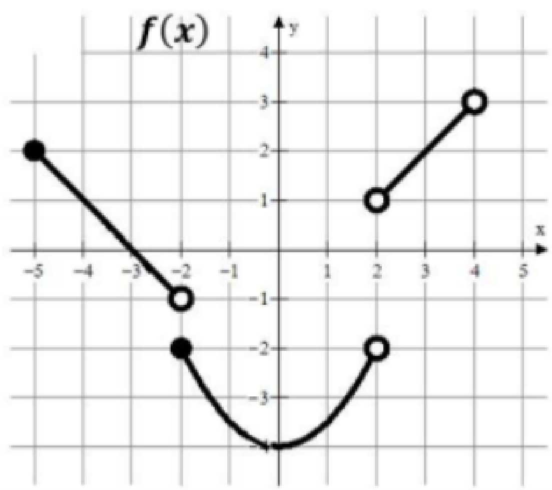
$f(0) =$

$f(4) =$

$f(-2) =$

$f(x) = 2$ when $x = ?$

$f(f(0)) =$



Piecewise Functions

For the piecewise function

$$f(x) = \begin{cases} 4 & x \leq 2 \\ 3x - 1 & x > 2 \end{cases}.$$

36) Evaluate $f(-1)$, $f(2)$ and $f(5)$

37) Solve the equation $f(x) = 4$

38) Graph $f(x)$

Section 2: Simplifying and Solving

Common Algebra Mistakes

The solution to each of the following equations contains at least one step (and possibly more) where an algebra mistake has been made. Identify and briefly explain the mistake(s), then re-solve the problems correctly.

1) $10x^2 + 7x = 12$
$x(10x + 7) = 12$
$x = 12$ and $10x + 7 = 12$
$x = 12$, $x = \frac{1}{2}$

2) $(x - 5)^2 = 16$
$x^2 - 25 = 16$
$x^2 = 41$
$x = \pm\sqrt{41}$

3) $x^3 = x^2$
Divide both sides by x^2
$x = 1$

4) $5x + 2x^{-1} = -11$
$\frac{5x+2}{x} = -11$
$5x + 2 = -11x$
$16x = -2$
$x = -\frac{1}{8}$

5) $2x^{-1} = 4$
$\frac{1}{2x} = 4$
$8x = 1$
$x = \frac{1}{8}$

6) $\sqrt{x^2 - 16} = 3$
$x - 4 = 3$
$x = 7$

7) $\frac{1}{x} + \frac{1}{2} = \frac{1}{3}$
$x + 2 = 3$
$x = 1$

8) $x = \sqrt{4}$
$x = \pm 2$

9) $\ln(x - 3) = 2$
$\ln x - \ln 3 = 2$
$\ln x = 2 + \ln 3$
$x = e^{2 + \ln 3}$
$x = e^2 \cdot e^{\ln 3}$
$x = 3e^2$

10) $e^{3 \ln x} = 27$
$3x = 27$
$x = 9$

Simplifying and Solving Equations
<u>Skills</u>: Factor expressions completely, simplify complex fractions, use these and other techniques to solve equations.

Simplify each by factoring completely.

11) $3x^3 - 5x^2 + 2x$

12) $4x^4 + 7x^2 - 36$

13) $5x^4 - 5y^4$

14) $x^3 - xy^2 + x^2y - y^3$

15) $-2(4x+1)^3(1-x) + 12(1-x)^2(4x+1)^2$

Simplify each by combining into a single fraction.

16) $\dfrac{1}{x+h} - \dfrac{1}{x}$

17) $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}$

18) $\dfrac{\frac{1}{3+x} - \frac{1}{3}}{x}$

19) $\dfrac{x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} + \dfrac{1}{\sqrt{1-x^2}}$

Solve each equation. Show all work to support your answer.

20) $2x^{-\frac{1}{3}} - 1 = 0$

21) $\left|\dfrac{x-3}{x+4}\right| = 5$

22) $2 - \dfrac{1}{x+1} = \dfrac{1}{x^2+x}$

23) Solve for z, $x\left(y^2 - z\right) = z\left(x^2 - y\right)$

Exponential and Logarithmic Functions (Simplifying and Solving)

Exponent Properties

$a^0 = 1$ $\quad (a^m)^n = a^{m\cdot n}$

$a^{-m} = \frac{1}{a^m}$ $\quad (ab)^m = a^m \cdot b^m$

$a^m \cdot a^n = a^{m+n}$ $\quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$\frac{a^m}{a^n} = a^{m-n}$ $\quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Logarithm Properties

$\log_b 1 = 0$

$\log_b (m \cdot n) = \log_b(m) + \log_b(n)$

$\log_b \left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$

$\log_b (m^n) = n \cdot \log_b(m)$

Exponential and Logarithmic functions are inverses of each other. That means they can be used to undo each other. Here are some useful relationships:

If $b^m = n$ the $\log_b(n) = m$ $\quad \log_b(b^a) = a$ $\quad \ln(e^a) = a$ $\quad b^{\log_b(a)} = a$ $\quad e^{\ln(a)} = a$

Simplify and write with positive exponents and/or roots where applicable.

24) $\left(-5x^3\right)^{-2}$

25) $\left(-\frac{3}{x^5}\right)^{-4}$

26) $\left(36x^{10}\right)^{\frac{1}{2}}$ *Be careful!

27) $\left(16x^{-2}\right)^{\frac{3}{4}}$

28) Show that $\frac{8^x}{2^{x+5}}$ simplifies to 2^{2x-5}

29) Show that $e^{2x+\ln 5}$ simplifies to $5e^{2x}$

Evaluate each expression. There should be no logarithms or exponents in your final answers.

30) $5^{\log_5 40}$

31) $3^{2+\log_3 5}$

32) $2\log_3 9$

33) $\log_4 192 - \log_4 3$

34) $\log_4 8$

35) $\ln \sqrt{e}$

Solve. Show all steps that lead to your answer.

36) $3^x = 0$

37) $e^x - x^2e^x = 0$

38) Show that $x = 0$ and $x = 1$ by solving $\log_9\left(x^2 - x + 3\right) = \frac{1}{2}$ for x

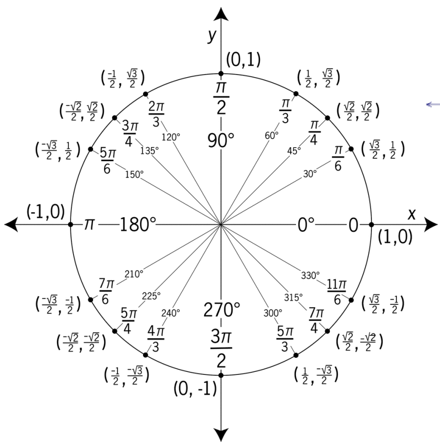
39) Show that $x = \frac{e}{e-1}$ by solving $\ln x - \ln(x-1) = 1$ for x

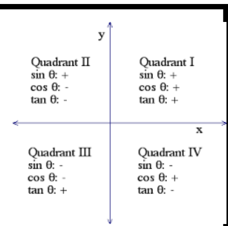
40) Show that $x = \log_3 2 + 4$ by solving $3^{x-2} = 18$ for x

41) Show that $x = \sqrt{e}$ by solving $\ln x^3 - \ln x^2 = \frac{1}{2}$ for x

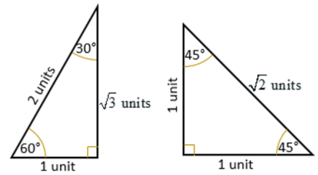
Section 3: Trigonometry

Evaluating Trig Functions





Special Triangles



	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Example:
$\cos\left(\frac{9\pi}{4}\right) =$

Solution:
θ in Quadrant 1
Ref Ang: $\hat{\theta} = \frac{\pi}{4}$
cos > 0 in Q1
$\cos\left(\frac{9\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Evaluate each trig function *without* a calculator. For each problem, state the quadrant that the angle terminates in (or axis angle terminates on), the reference angle (or NA if on axis), the sign of the trig function (+/-), then evaluate the trig function.

1) $\sin\left(\frac{5\pi}{6}\right) =$

2) $\tan\left(-\frac{\pi}{4}\right) =$

3) $\cos\left(\frac{21\pi}{4}\right) =$

4) $\csc\left(\frac{5\pi}{3}\right) =$

5) $\cot\left(\frac{13\pi}{3}\right) =$

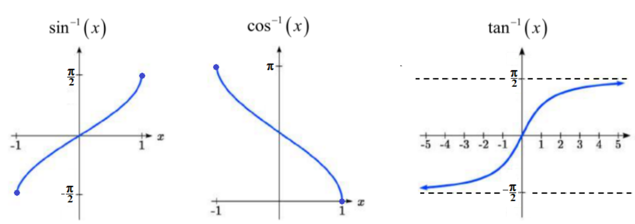
6) $\sin 11\pi =$

7) $\sec\left(-\frac{7\pi}{6}\right) =$

8) $\cos\left(-\frac{19\pi}{6}\right) =$

Inverse Trig Functions

Recall that inverse trig functions can be written as $\sin^{-1}(x)$ OR $\arcsin(x)$. Inverse trig functions have an input of a ratio and output an angle.



Evaluate each of the following. Remember: inverse trig functions are functions (so for one input they will have at most one output).

9) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

10) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

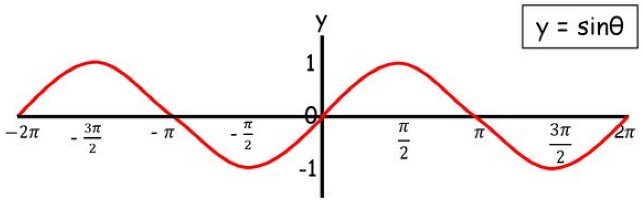
11) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

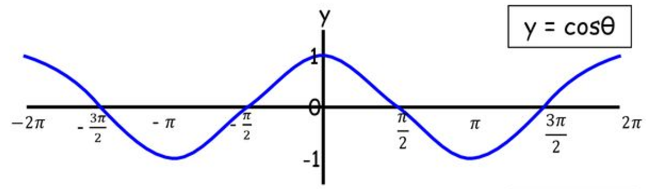
12) $\arccos(-1)$

13) $\arcsin(-0.5)$

14) $\arctan(-1)$

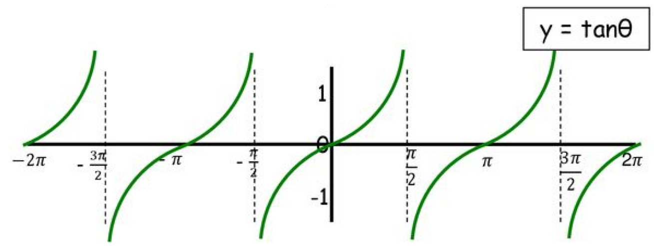
Graphing Trig Functions





Parent graphs for sine, cosine and tangent are given.

All other graphs can be obtained by applying transformations to the trig functions (as you would other functions).



Sketch *at least* two periods of each function. Label the axes and indicate the amplitude and period.

15) $y = \cos\left(\frac{1}{3}x\right)$

16) $y = 1 - \frac{1}{4}\sin 2x$

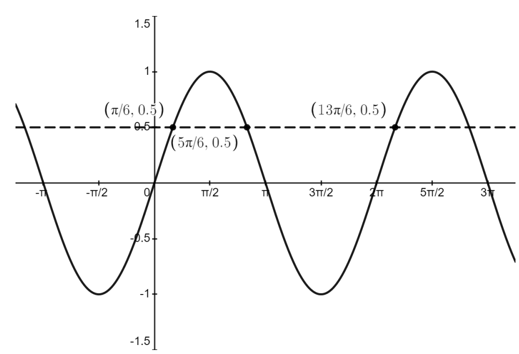
17) $y = \sin\left(x - \frac{\pi}{4}\right)$

18) $y = |\cos x|$

Solving Trig Equations

Example: Find all solutions for $\sin x = \frac{1}{2}$

Solution:
Find all solutions on the unit circle $[0, 2\pi)$, then add all coterminal angles by adding $2\pi k$ where k is an integer.
So, $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$
All solutions: $x = \frac{\pi}{6} + 2\pi k$ and $x = \frac{5\pi}{6} + 2\pi k$



Solve the equations given below on the interval $[0, 2\pi)$. Do not use a calculator.

19) $\cos\theta = -\frac{1}{2}$

20) $\sec\theta = \frac{2}{\sqrt{3}}$

21) $\tan^2 x = 1$

22) $\cos^2 x - \cos x \sin x = 0$

23) $\cot 3x = \sqrt{3}$

24) $\cos\left(\frac{\pi}{6}x\right) = 0$

Section 4: Miscellaneous

Implied Domain

Skill: Identify the domain of a function defined algebraically.

There are a few types of domain restrictions to consider when identifying a function's domain given the function algebraically. These are:

1. Cannot divide by zero.
2. Cannot take the even root of a negative number.
3. Cannot take the logarithm of anything ≤ 0.

1) $y = \sqrt{5-x}$

2) $y = \frac{x-2}{x+5}$

3) $y = \frac{1}{\sqrt{x-3}}$

4) $y = \log_4(x+1) - 3$

5) $y = x^{\frac{3}{2}}$

6) $y = x^{\frac{2}{3}}$

7) $y = x^{-1}$

8) $y = x^3 - 3x^2 + 3x - 2$

9) $y = \frac{1}{x^2-4}$

10) $y = \frac{x+3}{x^2+1}$

11) $y = \sqrt{e^x}$

12) $y = \frac{\ln(x+3)}{x-9}$

13) $y = (\sqrt{x})^2$

14) $y = \sqrt{x^2}$

15) Why are the domains for #13 & 14 different?

Solving Nonlinear Inequalities

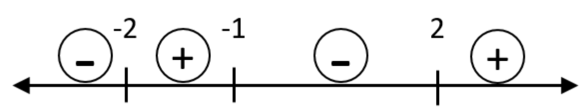
Skill: Solve nonlinear inequalities algebraically.

Example: Solve $\frac{x^2-4}{x+1} > 0$

Step 1: Find all values where $f(x) = 0$ and where $f(x)$ is undefined. *These are the two types of x-values where a function can possibly change sign.*

$f(x) = 0$ when $x^2 - 4 = 0$, $(x-2)(x+2) = 0$, therefore $x = 2$ and -2
$f(x)$ is undefined when $x + 1 = 0$ so $x = -1$

Step 2: Put these values on a number line and then evaluate each interval to create a sign chart.



Step 3: List all intervals that satisfy the inequality.

$(-2, -1)$ and $(2, \infty)$

16) $2x^2 - x - 3 < 0$

17) $e^x + xe^x > 0$

18) $\sin(2x) < 0$ on $(-\pi, \pi)$

19) $\cos\left(\frac{\pi}{3}x\right) > 0$ on $(0, 8)$

20) $\ln(x-1) > 0$

21) $\frac{\sqrt{x+2}}{x^2-9} < 0$

Section 5: Graphing Calculator Skills

The QR codes below will take you to videos reviewing usage of the TI-89 calculator. If you have a different model calculator please find a comparable video, or you can experiment on your own.

Create a Table of Values

It is important that you know how to create a table of values in order to evaluate a function. We will be using a table of values to evaluate limits numerically at the beginning of the year, and you may need to consult the table feature to get some points on a function as this will help identify an appropriate viewing window for the graph.

For assistance with the problems below please watch this video on the explanation of the TI-89 calculator functionality.

Answer the questions below using the table feature of your calculator. Round all answers to *exactly* three decimal places.

1) Create a table of values for the function $h(x) = x^4 - 2.3x^3 + 4$ and list the first five values starting at $x = -10$ and step of 2.

2) Create a table of values for the function $f(x) = 50\cos\left(\frac{x}{100}\right)$ and list the first five values starting at $x = 5$ and step of 0.25. *Make sure calculator in RADIANS

3) Change the independent variable in the table setup from auto to ask. Use a table to get the outputs listed below for $f(x) = \dfrac{x-2}{x^2-4}$.

4) Change the independent variable in the table setup from auto to ask. Use a table to get the outputs listed below for $g(x) = \dfrac{\sin(x)}{x}$.

Graph Functions and Find Attributes

It is important that you know how to enter functions into the equation editor, [y =] key on your calculator, view your function in an appropriate window, and find attributes of the function.

For assistance with the problems below please watch this video on the explanation of the TI-89 calculator functionality.

5) Graph $y = x + 4$ and $y = x^2 - 1$ and find the points of intersection.

State the points of intersection.

6) Graph $y = x^2 - 13$ in the standard viewing window. Change your window so that you can see all important attributes of the graph. State the components of your viewing window.

7) Graph $f(x) = (x+1)(x^2-2)(x+5)$.

Find the values of all relative maximums and minimums, and identify the location of each.

8) Graph $y = \sin\left(\frac{x}{2}\right)\cos(5x)$.

Find all x-intercepts on the interval $[0, 3]$. Make sure your calculator is in radian mode.